

# ACCURATE MODELS FOR MICROSTRIP COMPUTER-AIDED DESIGN

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## Summary

Very accurate and simple equations are presented for both single and coupled microstrip lines' electrical parameters, i.e. impedances, effective dielectric constants, and attenuation including the effect of anisotropy in the substrate. For the single microstrip the effects of dispersion and non-zero strip thickness are also included.

## Introduction

In microstrip design it is often observed that the physical circuit performance differs significantly from that theoretically calculated. This is due to factors such as attenuation, dispersion, and discontinuity effects, which again are functions of physical parameters and the actual circuit lay-out. The conclusion which may be drawn from this observation, is that optimum microstrip design should be based directly upon physical dimensions. This approach requires accurate models and for computer-aided design the models must also be in an easily calculable form.

The models presented in this paper represent partly completely new models, but also revisions and expansions of some which have been given earlier<sup>1,2</sup>. It has been a goal to obtain an accuracy which gives errors at least less than those caused by physical tolerances. The line models are based on theoretical data<sup>1,2</sup>, while the discontinuity models also are based on experimental data.

## Single microstrip

The model of the single microstrip line is based upon an equation for the impedance of microstrip in an homogeneous medium,  $Z_{01}$ , and an equation for the microstrip effective dielectric constant,  $\epsilon_e$ .

$$Z_{01}(u) = \frac{\eta_0}{2\pi} \ln \left[ \frac{f(u)}{u} + \sqrt{1 + \left(\frac{2}{u}\right)^2} \right] \quad (1)$$

$$f(u) = 6 + (2\pi - 6) \exp \left[ - \left( \frac{30.666}{u} \right)^{0.7528} \right] \quad (2)$$

Here  $\eta_0$  is wave impedance of the medium (376.73Ω in vacuum) and  $u$  is the strip width normalized with respect to substrate height ( $w/h$ ). The accuracy of this model is better than 0.01% for  $u \leq 1$  and 0.03% for  $u \leq 1000$ .

$$\epsilon_e(u, \epsilon_r) = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left( 1 + \frac{10}{u} \right)^{-a(u)} b(\epsilon_r) \quad (3)$$

$$a(u) = 1 + \frac{1}{49} \ln \frac{u^4 + (u/52)^2}{u^4 + 0.432} + \frac{1}{18.7} \ln \left[ 1 + \left( \frac{u}{18.1} \right)^3 \right] \quad (4)$$

$$b(\epsilon_r) = 0.564 \left( \frac{\epsilon_r - 0.9}{\epsilon_r + 3} \right)^{0.053} \quad (5)$$

The accuracy of this model is better than 0.2% at least for  $\epsilon_r \leq 128$  and  $0.01 \leq u \leq 100$ .

Compared to earlier equations, these give much better accuracy, in the order of 0.1% for both impedance and wavelength. They are also complete with respect to range of strip width, while earlier ones were given in two sets.

## Strip thickness correction

In correcting the above results for non-zero strip thickness, a method described by Wheeler<sup>3</sup> is used. However, some modifications in his equations have been made, which give better accuracy for narrow strips and for substrates with low dielectric constant.

For homogeneous media the correction is

$$\Delta u_1 = \frac{t}{\pi} \ln \left( 1 + \frac{4 \exp(1)}{t \coth^2 \sqrt{6.517} u} \right) \quad (6)$$

where  $t$  is the normalized strip thickness. For mixed media the correction is

$$\Delta u_r = \frac{1}{2} (1 + 1/\cosh \sqrt{\epsilon_r - 1}) \Delta u_1 \quad (7)$$

By defining corrected strip widths,  $u_1 = u + \Delta u_1$  and  $u_r = u + \Delta u_r$ , the effect of strip thickness may be included in the above equations:

$$Z_0(u, t, \epsilon_r) = Z_{01}(u_r) / \sqrt{\epsilon_e(u_r, \epsilon_r)} \quad (8)$$

$$\epsilon_{eff}(u, t, \epsilon_r) = \epsilon_e(u_r, \epsilon_r) \cdot [Z_{01}(u_1) / Z_{01}(u_r)]^2 \quad (9)$$

## Dispersion

As microstrip propagation is not purely TEM, both impedance and effective dielectric constant vary with frequency. Getsinger<sup>4</sup> has proposed the following model for dispersion in effective dielectric constant

$$\epsilon_{eff} = \epsilon_r - \frac{\epsilon_r - \epsilon_{eff}(0)}{1 + G(f/f_p)^2} \quad (10)$$

where  $f_p = Z_0 / (2u_0 h)$  may be regarded as an approximation to the first TE-mode cut-off frequency while  $G$  is a factor which is empirically determined. Getsinger gave an expression for  $G$  which fitted experimental data for alumina substrates, but this did not give correct results for other substrates. The following expression for  $G$  has been shown to give very good results for all types of substrates now in use:

$$G = \frac{\pi^2}{12} \frac{\epsilon_r - 1}{\epsilon_{eff}(0)} \sqrt{\frac{2\pi Z_0}{\eta_0}} \quad (11)$$

There is today not general agreement on a model for microstrip impedance dispersion. Based on a parallel-plate model and using the theory of dielectrics, the following equation was found.

$$Z_0(f) = Z_0(0) \sqrt{\frac{\epsilon_{eff}(0)}{\epsilon_{eff}(f)}} \cdot \frac{\epsilon_{eff}(f) - 1}{\epsilon_{eff}(0) - 1} \quad (12)$$

While we would accept theoretical objections to this model on the grounds that microstrip impedance is rather arbitrarily defined, it may be pointed out that it agrees very well with the calculations of Krage & Haddad<sup>5</sup> and that in the limit  $f \rightarrow \infty$  it is agreement with three of the definitions discussed by Bianco & al.<sup>6</sup>. It may also be noted that the model predicts a rather small but positive increase in impedance with frequency and that this would seem to fit experimental observations.

#### Coupled microstrips

The equations given below represent the first generally valid model of coupled microstrips with an acceptable accuracy. The model is based upon perturbations of the homogeneous microstrip impedance equation and the equations for effective dielectric constant. The equations have been validated against theoretically calculated data in the range  $0.1 < u < 10$  and  $g > 0.01$  ( $g$  is the normalized gap width ( $s/h$ )), a range which should cover that used in practise.

The homogeneous mode impedances are

$$Z_{01m}(u, g) = Z_{01}(u) / [1 - Z_{01}(u) \phi_m(u, g) / \eta_0] \quad (13)$$

where the index  $m$  is set to  $e$  for the even mode and  $o$  for the odd. The effective dielectric constants are

$$\epsilon_{em}(u, g, \epsilon_r) = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} F_m(u, g, \epsilon_r) \quad (14)$$

$$F_e(u, g, \epsilon_r) = [1 + \frac{10}{u(u, g)}]^{-a(u)b(\epsilon_r)} \quad (15)$$

$$F_o(u, g, \epsilon_r) = f_o(u, g, \epsilon_r) (1 + \frac{10}{u})^{-a(u)b(\epsilon_r)} \quad (16)$$

The modifying equations are as follows

$$\phi_e(u, g) = \phi(u) / \psi(g) [\alpha(g) u^m(g) + [1 - \alpha(g)] u^{-m(g)}] \quad (17)$$

$$\phi(u) = 0.8645 u^{0.172} \quad (18)$$

$$\psi(g) = 1 + \frac{g}{1.45} + \frac{g^{2.09}}{3.95} \quad (19)$$

$$\alpha(g) = 0.5 \exp(-g) \quad (20)$$

$$m(g) = 0.2175 + [4.113 + (\frac{20.36}{g})^6]^{-0.251} + \frac{1}{323} \ln \frac{g^{10}}{1 + (\frac{g}{13.8})^{10}} \quad (21)$$

$$\phi_o(u, g) = \phi_e(u, g) - \frac{\theta(g)}{\psi(g)} \exp[\beta(g) u^{-n(g)} \ln u] \quad (22)$$

$$\theta(g) = 1.729 + 1.175 \ln(1 + \frac{0.627}{g + 0.327 g^{2.17}}) \quad (23)$$

$$\beta(g) = 0.2306 + \frac{1}{301.8} \ln \frac{g^{10}}{1 + (\frac{g}{3.73})^{10}} + \frac{1}{5.3} \ln(1 + 0.646 g^{1.175}) \quad (24)$$

$$n(g) = \{ \frac{1}{17.7} + \exp[-6.424 - 0.76 \ln g - (\frac{g}{0.23})^5] \} \cdot \ln \frac{10 + 68.3 g^2}{1 + 32.5 g^{3.093}} \quad (25)$$

$$u(u, g) = g \exp(-g) + u \cdot \frac{20 + g^2}{10 + g^2} \quad (26)$$

$$f_o(u, g, \epsilon_r) = f_{o1}(g, \epsilon_r) \exp[p(g) \ln u + q(g) \sin(\pi \frac{\ln u}{\ln 10})] \quad (27)$$

$$p(g) = \exp(-0.745 g^{0.295}) / \cosh(g^{0.68}) \quad (28)$$

$$q(g) = \exp(-1.366 - g) \quad (29)$$

$$f_{o1}(g, \epsilon_r) = 1 - \exp[-0.179 g^{0.15} - \frac{0.328 g^r(g, \epsilon_r)}{\ln \exp(1) + (g/7)^{2.8}}] \quad (30)$$

$$r(g, \epsilon_r) = 1 + 0.15 [1 - \frac{\exp(1 - (\epsilon_r - 1)^2 / 8.2)}{1 + g^{-6}}] \quad (31)$$

When the development of the above equations was done (1) and (2) were not available. Instead a simpler equation for homogeneous microstrip impedance with a maximum error of 0.3% for  $u > 0.06$  was used:

$$Z_{01}(u) = \eta_0 / (u + 1.98 u^{0.172}) \quad (32)$$

The errors in the even and odd mode impedances where then found to be less than 0.8% and less than 0.3% for the wavelengths. The above model does not include the effect of non-zero strip thickness or asymmetry. It is a goal at ELAB to include these factors, although the parameter range probably will have to be restricted to the strictly practically useful in regard of the complexity of the above equations.

Dispersion is also not included presently. Getsinger<sup>7</sup> has proposed modifications to his single strip dispersion model, but unfortunately it is easily shown that the results are asymptotically wrong for extreme values of gap width. However, it is possible to modify the Getsinger dispersion model so that the asymptotic values are satisfied for coupled lines, but the details remain to be worked out.

#### Attenuation

The strip capacitive quality factor is

$$Q_d = \frac{(1 - q) + q \epsilon_r}{(1 - q) / Q_A + q \epsilon_r / Q_s} \quad (33)$$

where  $q$  is the mixed dielectric filling fraction,  $Q_A$  is the dielectric quality factor of the upper half space medium, and  $Q_s$  is the substrate quality factor.

The strip inductive quality factor is approximated by<sup>8</sup>:

$$Q_c = \frac{\pi Z_{01} h f}{R_s c} \cdot \frac{u}{K} \quad (34)$$

where  $R_s$  is the skin resistance and  $K$  is the current distribution factor. Surface roughness will increase the skin resistance<sup>1</sup>.

$$R_s(\Delta) = R_s(0) [1 + \frac{2}{\pi} \operatorname{Arctg} \tan 1.4 (\frac{\Delta}{\delta})^2] \quad (35)$$

where  $\delta$  is the skin depth and  $\Delta$  is the rms surface roughness.

For the single microstrip current distribution factor we have found

$$K = \exp(-1.2 \frac{Z_{01}(u)}{\eta_0} 0.7). \quad (36)$$

to be a very good approximation to the current distribution factor due to Pucel et al. <sup>9</sup> provided that the strip thickness exceeds three skin depths.

The microstrip quasi-TEM mode quality factor,  $Q_0$ , is now given by

$$\frac{1}{Q_0} = \frac{1}{Q_d} + \frac{1}{Q_c} \quad (37)$$

and the attenuation factor becomes

$$\alpha \left[ \frac{dB}{m} \right] = \frac{20\pi}{\ln 10} \frac{G}{Q_0 f \epsilon_{eff}} \quad (38)$$

The above loss equations are also valid for coupled microstrips <sup>8</sup>, provided that the dielectric filling factor, homogenous impedance, and current distribution factor of the actual mode are used. Presently, no equations are available for the odd and even mode current distribution factors. Except for very tight coupling, however, the following approximation gives good results

$$K_e = K_o = \exp[-1.2 \frac{Z_{01} \epsilon_{eff}}{2\eta_0} 0.7] \quad (39)$$

#### Anisotropy

From recent work <sup>10,11</sup> it may be shown that the above models are also valid with anisotropic substrates by defining an isotropyzied substrate where

$$h_{eq} = h \sqrt{\epsilon_x / \epsilon_y} \quad (40)$$

$$\epsilon_{eq} = \sqrt{\epsilon_x \epsilon_y} \quad (41)$$

This method requires that one of the substrate's principal axes is parallel to the substrate (x-axis) and the other normal to the substrate (y-axis). The strip dimensions remain unchanged, but are normalized with respect to  $h_{eq}$ . The procedure for utilizing the above models then proceed through calculation of the mode capacitances,  $C_m$ ,

$$C_m = 1 / (v_{eqm} Z_{eqm}) \quad (42)$$

Here  $v_{eqm}$  is the mode phase velocity and  $Z_{eqm}$  the mode impedance on the isotropyzied substrate. Calculating the homogeneous mode capacitance,  $C_{m1}$ , the impedance and effective dielectric constant on the anisotropic substrate are then

$$Z_{0m} = 1 / (\epsilon \sqrt{C_m C_{m1}}) \quad (43)$$

$$\epsilon_{effm} = C_m / C_{m1} \quad (44)$$

The inductive quality factor of the aniso-

tropic substrate, may be directly calculated with the above equations, while the substrate quality factor has to be modified:

$$Q_s = 2 \left( \frac{1}{Q_{sx}} + \frac{1}{Q_{sy}} \right)^{-1} \quad (45)$$

#### Discontinuities

Due to the illness, modelling of microstrip discontinuities could unfortunately not be finished to meet the digest deadline. The final paper presented at the symposium will give models for the microstrip open end, the gap, compensated right-angle bend, and the T-junction.

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